3D Geometric Transformation

3D computer graphics (in contrast to 2D computer graphics) are graphics that use a three-dimensional representation of geometric data (often Cartesian) that is stored in the computer for the purposes of performing calculations and rendering 2D images. Such images may be stored for viewing later or displayed in real-time.

Contents

- Translation
- Scaling
- Rotation
- Rotations with Quaternions

Transformation in 3D

- Transformation Matrix

$$\begin{bmatrix} A & D & G & J \\ B & E & H & K \\ C & F & I & L \\ 0 & 0 & 0 & S \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} & 3\times 3 & & 3\times 1 \\ & & & \\ \hline & 1\times 3 & & 1\times 1 \end{array}\right]$$

3×3 : Scaling, Reflection, Shearing, Rotation

3×1 : Translation

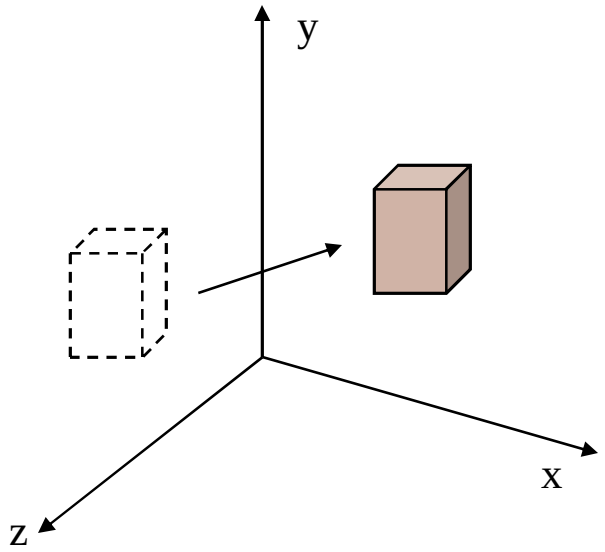
1×1 : Uniform global Scaling

1×3 : Homogeneous representation

3D Translation

- Translation of a Point

$$x' = x + t_x, \quad y' = y + t_y, \quad z' = z + t_z$$

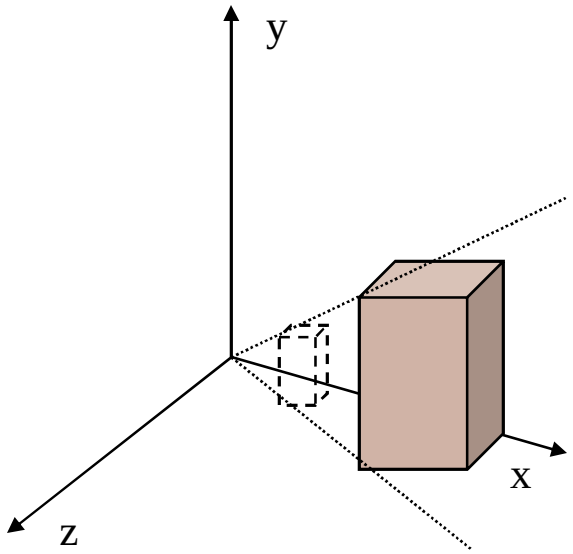


$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Scaling

- Uniform Scaling

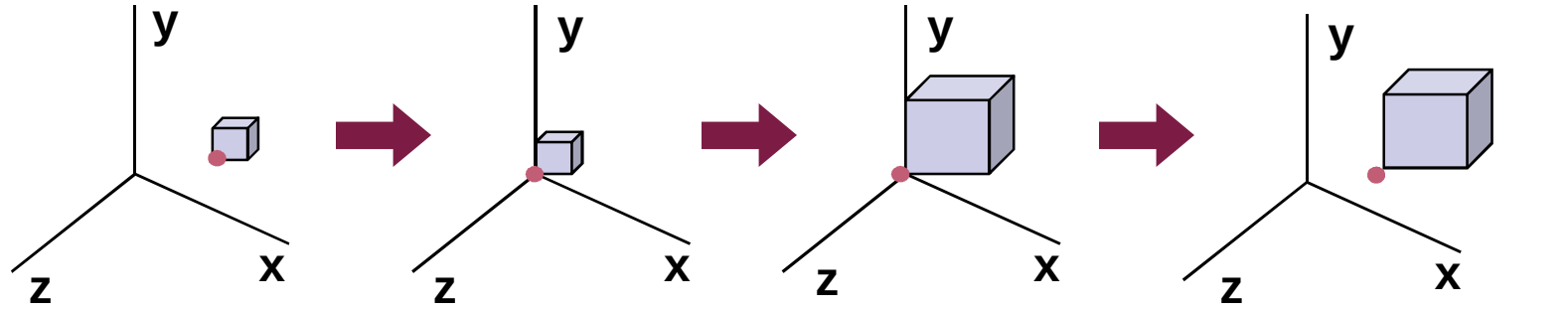
$$x' = x \cdot s_x, \quad y' = y \cdot s_y, \quad z' = z \cdot s_z$$



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Relative Scaling

- Scaling with a Selected Fixed Position



Original position | Translate | Scaling | Inverse Translate

$$T(x_f, y_f, z_f) \cdot S(s_x, s_y, s_z) \cdot T(-x_f, -y_f, -z_f) = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x_f \\ 0 & 1 & 0 & y_f \\ 0 & 0 & 1 & z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_f \\ 0 & 1 & 0 & -y_f \\ 0 & 0 & 1 & -z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Rotation

- Coordinate-Axes Rotations
  - X-axis rotation
  - Y-axis rotation
  - Z-axis rotation
- General 3D Rotations
  - Rotation about an axis that is parallel to one of the coordinate axes
  - Rotation about an arbitrary axis

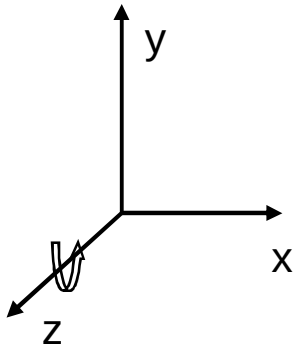
Coordinate-Axes Rotations

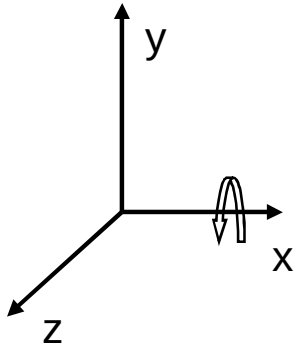
- Z-Axis Rotation
- X-Axis Rotation
- Y-Axis Rotation

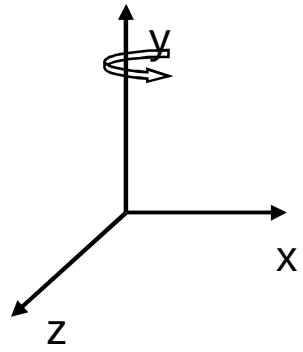
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

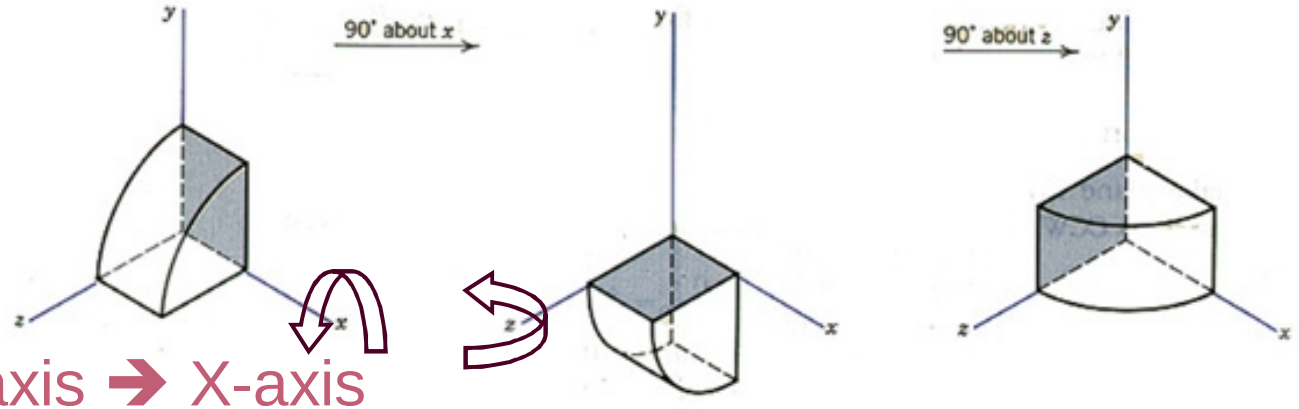




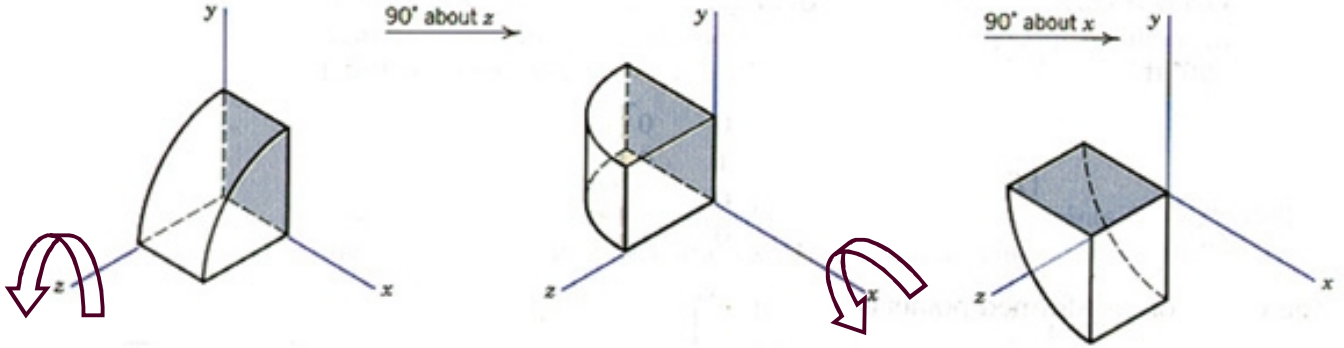


Order of Rotations

- Order of Rotation Affects Final Position
  - X-axis ➔ Z-axis



  - Z-axis ➔ X-axis



Application

A **3D** (**three-Dimensional**) **film** or **S3D** (**stereoscopic 3D**) **film** is a motion picture that enhances the illusion of depth perception. Derived from stereoscopic photography, a regular motion picture camera system is used to record the images as seen from two perspectives (or computer-generated imagery generates the two perspectives in post-production), and special projection hardware and/or eyewear are used to provide the illusion of depth when viewing the film.

Scope of Research

The first widely available commercial application of human virtu al models appeared in 1998 on the Lands' End web site. The hum an virtual models were created by the company My Virtual Mode l Inc. and enabled users to create a model of themselves and try o n 3D clothing. There are several modern programs that allow for the creation of virtual human models (Poser being one example).